Answers :

(1) (i) Gradient of $AB = \frac{3-0}{6-2} = \frac{3}{4}$

Therefore, gradient of $BC = -\frac{4}{3}$ (as $\angle ABC = 90°$)

(ii) $\frac{(2k+1)-3}{k-6} = -\frac{4}{3}$

$\frac{2k-2}{k-6} = -\frac{4}{3}$

$6k - 6 = -4k + 24$

$10k = 30$

$k = 3$

Coordinates of $C = (3, 7)$

(iii) since M is the mid-point of AB,

Coordinates of $M = (\frac{6+2}{2}, \frac{3+0}{2})$

$= (4, \frac{3}{2})$

(iv) Gradient of $DM = -\frac{4}{3}$ (since $DM \parallel BC$)

Therefore equation of DM is, $y - 4 = -\frac{4}{3}(x - \frac{3}{2})$

$y - 4 = -\frac{4}{3}x + 2$

$y = -\frac{4}{3}x + 6$

(v) solving $y = -\frac{4}{3}x + 6$ and $y = 16 - 8x$,

$-\frac{4}{3}x + 6 = 16 - 8x$

$8x - \frac{4}{3}x = 10$

$\frac{20}{3}x = 10$

$x = \frac{3}{2}$

$y = 16 - 8\left(\frac{3}{2}\right) = 4$

Therefore coordinates of $D = (\frac{3}{2}, 4)$

(vi) Area of quadrilateral $ABCD = \frac{1}{2}\begin{vmatrix} 2 & 6 & 3 & \frac{3}{2} & 2 \\ 0 & 3 & 7 & 4 & 0 \end{vmatrix}$

$= \frac{1}{2}\{(6 + 42 + 12 + 0) - (0 + 9 + \frac{21}{2} + 8)\}$

$= \frac{1}{2}\left(\frac{65}{2}\right) = 16\frac{1}{4}$ square units