

Statistics (Standard Deviation)

- (1) Standard deviation for ungrouped data – It is a measure of the spread of its values. It is defined as the square root of the variance.

$$\text{Standard deviation, } \sigma = \sqrt{\frac{\sum(x_i - \mu)^2}{n}}$$

$$\text{Or} = \sqrt{\frac{\sum x_i^2}{n} - \mu^2}$$

Where x_i is the available data and μ is the mean.

For example : Find the standard deviation from the given set values : 5 7 9 10 12 and 13.

$$\mu = \frac{\sum x_i}{n} = \frac{5 + 7 + 9 + 10 + 12 + 13}{6} = 9.5$$

$$\sigma = \sqrt{\frac{\sum x_i^2}{n} - \mu^2} = \sqrt{\frac{5^2 + 7^2 + 9^2 + 10^2 + 12^2 + 13^2}{6} - (9.5)^2}$$
$$= 2.99$$

The standard deviation of a frequency distribution without class intervals is given as,

$$\sigma = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \mu^2}, \text{ where } f_i \text{ is the frequency of the value of } x_i .$$

For example:

Age	14	15	16	17	18
Frequency	3	4	6	5	2

$$\mu = \frac{(3 \times 14) + (4 \times 15) + (6 \times 16) + (5 \times 17) + (2 \times 18)}{3 + 4 + 6 + 5 + 2}$$
$$= 15.95$$

$$\sigma = \sqrt{\frac{3 \times 14^2 + 4 \times 15^2 + 6 \times 16^2 + 5 \times 17^2 + 2 \times 18^2}{20} - (15.95)^2}$$
$$= 1.20$$

- (2) Standard deviation for grouped data – this is with class intervals.

$$\sigma = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \mu^2}, \text{ where } f_i \text{ is the frequency of the class interval and } x_i \text{ is the mid-value of the class interval.}$$

For example :

Time (t mins)	$0 < t \leq 20$	$20 < t \leq 40$	$40 < t \leq 60$	$60 < t \leq 80$
Frequency	4	6	5	3
Mid-value	10	30	50	70

$$\mu = \frac{(4 \times 10) + (6 \times 30) + (5 \times 50) + (3 \times 70)}{4 + 6 + 5 + 3}$$
$$= 37.78$$

$$\sigma = \sqrt{\frac{4 \times 10^2 + 6 \times 30^2 + 5 \times 50^2 + 3 \times 70^2}{4 + 6 + 5 + 3} - (37.78)^2}$$
$$= 20.15$$

Try these questions :

(1) Find the mean and standard deviation of this set of numbers: 4 6 7 9 11. If each of the numbers in the set is increased by 2, find the new mean and standard deviation. Comment on the results.

(2) The mass of 30 cookies were weighed to nearest grams. Find the mean mass and the standard deviation.

Mass (x)	$20 \leq x \leq 22$	$23 \leq x \leq 25$	$26 \leq x \leq 28$	$29 \leq x \leq 31$	$32 \leq x \leq 34$
Frequency(f)	3	6	11	8	2

